

MAT-207: Discrete Mathematics – Assignment 2

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- 1) Find the decimal equivalent to the following binary numbers: 10101, 100101, and 11010. Given a binary number, how would you decide whether it is an even number or an odd number?

a) 10101

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 & \\ 16 & + & 4 & + & 1 & \end{array}$$

$$16 + 4 + 1 = 21$$

b) 100101

$$\begin{array}{cccccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 32 & + & & 4 & + & 1 \end{array}$$

$$32 + 4 + 1 = 37$$

c) 11010

$$\begin{array}{cccccc} 1 & 1 & 0 & 1 & 0 & \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 & \\ 16 + 8 & + & & 2 & & \end{array}$$

$$16 + 8 + 2 = 26$$

The first digit of the binary number (2^0) will determine if a number is an odd or even value,

- 2) Find the binary representation (using both forward and backward methods) of the following decimal numbers: 28, 43, 100, and 81.

<p>28 12 4</p> <p>= 16 + 8 + 4</p> <p>= $2^4 + 2^3 + 2^2$</p> <p>= <u>1</u> <u>1</u> <u>1</u> <u>0</u> <u>0</u></p>	$\begin{array}{r} 2 \overline{)28} \quad 0 \\ \underline{2 \ 14} \quad 0 \\ 2 \overline{)7} \quad 1 \\ \underline{2 \ 3} \quad 1 \\ 2 \overline{)1} \quad 1 \\ \underline{ 0} \end{array}$
<p>100 32 4</p> <p>= 64 + 32 + 4</p> <p>= $2^6 + 2^5 + 2^2$</p> <p>= <u>1</u> <u>1</u> <u>0</u> <u>0</u> <u>1</u> <u>0</u> <u>0</u></p>	$\begin{array}{r} 2 \overline{)100} \quad 0 \\ \underline{2 \ 50} \quad 0 \\ 2 \overline{)25} \quad 1 \\ \underline{2 \ 12} \quad 0 \\ 2 \overline{)6} \quad 0 \\ \underline{2 \ 3} \quad 1 \\ 2 \overline{)1} \quad 1 \\ \underline{ 0} \end{array}$

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$43 = 32 + 8 + 2 + 1$ $= 2^5 + 2^3 + 2^1 + 2^0$ $= \underline{101011}$ <div style="display: flex; align-items: center; justify-content: center;"> <div style="text-align: right; margin-right: 10px;"> $\begin{array}{r} 2 \overline{)43} \quad 1 \\ 2 \overline{)21} \quad 1 \\ 2 \overline{)10} \quad 0 \\ 2 \overline{)5} \quad 1 \\ 2 \overline{)2} \quad 0 \\ 2 \overline{)1} \quad 1 \\ \hline 0 \end{array}$ </div> <div style="text-align: left; margin-left: 10px;"> \uparrow </div> </div>	$81 = 64 + 16 + 1$ $= 2^6 + 2^4 + 2^0$ $= \underline{1010001}$ <div style="display: flex; align-items: center; justify-content: center;"> <div style="text-align: right; margin-right: 10px;"> $\begin{array}{r} 2 \overline{)81} \quad 1 \\ 2 \overline{)40} \quad 0 \\ 2 \overline{)20} \quad 0 \\ 2 \overline{)10} \quad 0 \\ 2 \overline{)5} \quad 1 \\ 2 \overline{)2} \quad 0 \\ 2 \overline{)1} \quad 1 \\ \hline 0 \end{array}$ </div> <div style="text-align: left; margin-left: 10px;"> \uparrow </div> </div>
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- 3) How many binary digits are in the number $2^{128} + 1$?

A positive integer N has k bits when $2^{k-1} \leq N \leq 2^k - 1$ and can be written in a form of logarithm in base 2 as:

$$k = \lfloor \log_2(N) \rfloor + 1$$

Where k , the number of bits, is equal to floor of the logarithm of base 2 of N , which is the highest power of two in the binary representation of N , and $+1$ takes the exponent to the next higher of two which essentially accounts for the 2^0 th place of your binary power.

We can apply the equation below.

$$k = \lfloor \log_2(2^{128} + 1) \rfloor + 1$$

$$k = 129 \text{ digits}$$

Our answer is 129 digits.

- 4) How many binary digits are in the number $2^{128} - 1$?

We can apply the same procedure as shown in (3).

$$k = \lfloor \log_2(N) \rfloor + 1$$

$$k = \lfloor \log_2(2^{128} - 1) \rfloor + 1$$

$$k = 129 \text{ digits}$$

- 5) Translate each of the following decimal numbers into a base 3 representation: 13, 15, 21, 27, 30, and 80.

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13

$$\begin{array}{r} 3 \overline{) 13} \quad 1 \uparrow \\ 3 \overline{) 4} \quad 1 \uparrow \\ 3 \overline{) 1} \quad 1 \uparrow \\ \hline 0 \end{array}$$

Answer: $(111)_3$

15

$$\begin{array}{r} 3 \overline{) 15} \quad 0 \uparrow \\ 3 \overline{) 5} \quad 2 \uparrow \\ 3 \overline{) 1} \quad 1 \uparrow \\ \hline 0 \end{array}$$

Answer: $(120)_3$

21

$$\begin{array}{r} 3 \overline{) 21} \quad 0 \uparrow \\ 3 \overline{) 7} \quad 1 \uparrow \\ 3 \overline{) 2} \quad 2 \uparrow \\ \hline 0 \end{array}$$

Answer: $(210)_3$

27

$$\begin{array}{r} 3 \overline{) 27} \quad 0 \uparrow \\ 3 \overline{) 9} \quad 0 \uparrow \\ 3 \overline{) 3} \quad 0 \uparrow \\ 3 \overline{) 1} \quad 1 \uparrow \\ \hline 0 \end{array}$$

Answer: $(1001)_3$

30

$$\begin{array}{r} 3 \overline{) 30} \quad 0 \uparrow \\ 3 \overline{) 10} \quad 1 \uparrow \\ 3 \overline{) 3} \quad 0 \uparrow \\ 3 \overline{) 1} \quad 1 \uparrow \\ \hline 0 \end{array}$$

Answer: $(1010)_3$

80

$$\begin{array}{r} 3 \overline{) 80} \quad 2 \uparrow \\ 3 \overline{) 26} \quad 2 \uparrow \\ 3 \overline{) 8} \quad 2 \uparrow \\ 3 \overline{) 2} \quad 2 \uparrow \\ \hline 0 \end{array}$$

Answer: $(2222)_3$

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- 6) What can you say about the base 3 representation of a decimal number that when divided by 9 leaves a remainder of 7?

We can express this question into the equation below:

$$x = 9n + 7; n = 1, 2, 3, \dots$$

Here is a range from $n = 1$ to $n = 7$.

Expression	x in base 10	x In base 2
$x = 9(1) + 7$	16	121
$x = 9(2) + 7$	25	221
$x = 9(3) + 7$	34	1021
$x = 9(4) + 7$	43	1121
$x = 9(5) + 7$	52	1221
$x = 9(6) + 7$	61	2021
$x = 9(7) + 7$	70	2121

The first two digits (2^0 and 2^1) have the same value of $(21)_3$ while the third digit (2^2) and onwards displays a normal incrementation in base 3.

- 7) Find the base 5 representation of the following binary numbers: 110010_2 , using backward method.

① Convert base 2 to decimal.

$$\begin{aligned}
 110010_2 &= 2^5 + 2^4 + 2^1 \\
 &= 16 + 8 + 2 \\
 &= 26
 \end{aligned}$$

② Convert to base 5.

$$\begin{array}{r}
 5 \overline{) 26} \\
 \underline{5 } \\
 5 \\
 \underline{5 } \\
 1
 \end{array}$$

0

Answer: $(101)_5$

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8) Convert 21022_3 to base 6 representation, using forward method.

① Convert to decimal.

$$\begin{aligned} & 2 \quad 1 \quad 0 \quad 2 \quad 2_3 \\ & = (2 \cdot 3^4) + (1 \cdot 3^3) + (0 \cdot 3^2) + (2 \cdot 3^1) + (2 \cdot 3^0) \\ & = 197 \end{aligned}$$

② Convert decimal to base 6.

$$197 = \overset{161}{36} + \overset{125}{36} + \overset{89}{36} + \overset{53}{36} + \overset{17}{36} + 6 + 6 + 1 + 1 + 1 + 1$$

$$= (5 \cdot 6^2) + (2 \cdot 6^1) + (5 \cdot 6^0)$$

$$= (5 \ 2 \ 5)_3$$

$$6^0 = 1$$

$$6^1 = 6$$

$$6^2 = 36$$

$$6^3 = 216$$

$$6^4 = 1296$$