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Mat-213: Assignment 1

1. Use the Euclidean Algorithm to find the GCD of...

1.1 123 and 321

$$\gcd(123, 321)$$

; Euclidean Algorithm.

$$321 = 123 \cdot 2 + 75$$

$$123 = 75 \cdot 1 + 48$$

$$75 = 48 \cdot 1 + 27$$

$$48 = 27 \cdot 1 + 21$$

$$27 = 21 \cdot 1 + 6$$

$$21 = 6 \cdot 3 + 3$$

$$6 = 3 \cdot 2 + 0$$

$$\text{Therefore, } \gcd(123, 321) = \underline{\underline{3}}$$

1.2 247 & 736

$$\gcd(736, 247)$$

; Euclidean Algorithm.

$$736 = 247 \cdot 2 + 242$$

$$247 = 242 \cdot 1 + 5$$

$$242 = 5 \cdot 48 + 2$$

$$5 = 2 \cdot 2 + 1$$

$$2 = 1 \cdot 2 + 0$$

$$\text{Therefore, } \gcd(736, 247) = \underline{\underline{1}}$$

1.3 523 & 319

$$\gcd(523, 319)$$

; Euclidean Algorithm.

$$523 = 319 \cdot 1 + 204$$

$$319 = 204 \cdot 1 + 115$$

$$204 = 115 \cdot 1 + 89$$

$$115 = 89 \cdot 1 + 26$$

$$89 = 26 \cdot 3 + 11$$

$$26 = 11 \cdot 2 + 4$$

$$11 = 4 \cdot 2 + 3$$

$$4 = 3 \cdot 1 + 1$$

$$3 = 1 \cdot 3 + 0$$

$$\text{Therefore, } \gcd(523, 319) = \underline{\underline{1}}$$

2. Find λ and μ so that $\lambda \cdot 34 + \mu \cdot 21 = 1$

$$\lambda \cdot 34 + \mu \cdot 21 = 1$$

Rewrite as...

$$34\lambda + 21\mu = \gcd(34, 21)$$

(i) Find gcd via Euclidean Algo. (ii) Rewrite...

$$34 = 21 \cdot 1 + 13$$

$$21 = 13 \cdot 1 + 8$$

$$13 = 8 \cdot 1 + 5$$

$$8 = 5 \cdot 1 + 3$$

$$5 = 3 \cdot 1 + 2$$

$$3 = 2 \cdot 1 + 1$$

$$2 = 1 \cdot 2 + 0$$

Therefore, $\gcd(34, 21) = 1$

(iii) Finding the solution.

Since, $34\lambda + 21\mu = 1$...

$$3 - 2(1) = 1$$

$$3 - (5 - 3(1)) = 1$$

$$3 - 5 + (1)3 = 1$$

$$2(3) - 5 = 1$$

$$2(8 - 5(1)) - 5 = 1$$

$$2 \cdot 8 - 2(5) - 5 = 1$$

$$2 \cdot 8 - 3(5) = 1$$

| | |
|----------------|---|
| Therefore, | |
| $\lambda = -8$ | ☺ |
| $\mu = 13$ | |

$$2 \cdot 8 - 3(13 - 8(1)) = 1$$

$$2 \cdot 8 - 3 \cdot 13 + 3 \cdot 8 = 1$$

$$5(8) - 3 \cdot 13 = 1$$

$$5(21 - 13(1)) - 3 \cdot 13 = 1$$

$$5 \cdot 21 - 5 \cdot 13 - 3 \cdot 13 = 1$$

$$5 \cdot 21 - 8(13) = 1$$

$$5 \cdot 21 - 8(34 - 21) = 1$$

$$5 \cdot 21 - 8 \cdot 34 + 8 \cdot 21 = 1$$

$$-8 \cdot 34 + 13 \cdot 21 = 1$$

So, $34(-8) + 21(13) = 1$

3. Show that there are λ and μ such that $\lambda \cdot 111 + \mu \cdot 99 = 1$

$$\lambda \cdot 111 + \mu \cdot 99 = 1 = 111\lambda + 99\mu = \gcd(111, 99)$$

(i) Find gcd via Euclidean Algo. (ii) Since the GCD of 111 & 99

$$111 = 99 \cdot 1 + 12$$

$$99 = 12 \cdot 8 + 3$$

$$12 = 3 \cdot 4 + 0$$

Therefore, $\gcd(111, 99) = 3$

is 3 and for any $\lambda, \mu \in \mathbb{Z}$,

$111\lambda + 99\mu$ must be a multiple of 3 and that

1 is not divisible by 3,

there is no λ and μ that can exist!