

MAT-207: Discrete Mathematics – Assignment 1

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1) **How many different kinds of bags of 5 billiard balls can there be? (No cue ball, 15 balls in total)**

Here, we can use the multiplication principle in a form of “unordered selections”.

- a) Calculate how many different ways can we combine the 15 balls, which is by using the product rule.

$$15 \times 14 \times 13 \times 12 \times \dots \text{ or } 15!$$

- b) Since, we are interested in 5 balls only, we will divide with the factorial of the difference of the number of total objects (15) and the number of objects chosen (5) which is 10!.

$$\frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times \dots}{10 \times 9 \times \dots} \quad \text{or} \quad \frac{15!}{10!}$$

- c) And because we do not care about the order of the balls, we will divide it by how many ways the balls can be in order which is 5!.

$$\frac{15!}{10!} \times \frac{1}{5!} = 3003 \text{ bags}$$

- d) These steps form a formula called the “Combination Formula” where n is total number of objects and r is the number of objects chosen. This can be written in the following notations.

$$\frac{n!}{r!(n-r)!} \quad \text{or} \quad \binom{n}{r} \quad \text{or} \quad {}_n C_r$$

- e) The final answer is that there are 3,003 ways to combine 5 billiard balls, thus, 3,003 different bags.

2) **How many anagrams of MISSISSIPPI are there?**

As Anagrams are essentially ways to rearrange letters of a word to another word, we can find the number of anagrams by using the “ordered selection” or the “Permutation Formula” to find the answer.

- a) The word “MISSISSIPPI” has 11 letters means we can have 11! ways to rearrange these letters.

$$11 \times 10 \times 9 \times 8 \times 7 \times 6 \times \dots = 11!$$

- b) There are 4 I's, 4 S's and 2 P's, interchanging between those letters will not result in a new permutation, thus, we divide 11! by 4! × 4! × 2!

$$\frac{11!}{4! \times 4! \times 2!} = \frac{39,916,800}{24 \times 24 \times 2} = 34,650$$

- c) Our final answer is 34,650 anagrams.

3) **There are 6 different routes to fly from New York to Denver and 7 different routes to fly from Denver to San Francisco. How many routes can it be to fly from New York to San Francisco with a stop in Denver?**

a) Here, we are finding all possibilities of routes from New York to San Francisco, we would first need to identify the independent choices.



b) We then identify the number of options for each choice.



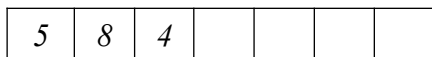
c) Finally, we use the multiplication principle by multiplying the two numbers to get the final answer.

$$6 \times 7 = 42 \text{ combinations of the two flights}$$

4) **How many different seven-digit phone numbers are there that begin with 584 and contain no zero? How many phone numbers are there that begin with 584 and contain at least one zero?**

First case: 7-digit phone numbers that begin with 584 and contain no zeros.

a) Identify the independent choices – there are 4 choices.



b) Determine all the possible options for each digit, since we do not include zero, there will only be 9 possible options for each digit (1 – 9)

$$9 \times 9 \times 9 \times 9$$

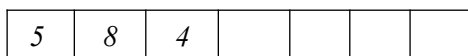
c) Apply the multiplication principle to find the number of arrangements of the digits.

$$9^4 = 9 \times 9 \times 9 \times 9 = 6,561$$

d) Final answer: 6,561 phone numbers.

Second case: Phone numbers that begin with 584 and contain at least one zero?

e) Identify the independent choices – there are 4 digits



f) Find the number of all possible phone numbers

$$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$$

g) Find the amount of phone numbers with no zeros (number taken from d.)

$$9^4 = 6,561$$

- h) Find numbers with at least one zero by subtracting 10^4 by 9^4 .

$$10^4 - 9^4 = 3,439$$

- i) Our final answer is 3,439 phone numbers contain at least one zero.

5) **How many ways to create a password from four English alphabet?**

- Identify the 4 independent choices
- Determine all possible letters / choices for each choice. In this context, we will have 26 possible letters for each independent choice. (A – Z)
- Use the “Permutation Formula” to calculate ways to make a four-letter password.

$${}_n P_r = \frac{n!}{(n-r)!} = \frac{26!}{(26-4)!} = 358,800$$

6) **Find the number of bit strings of size 10 that:**

6.1) Contain four 1’s

Use the combinations formula to calculate the number of bit strings that contain at least four 1’s.

$${}_n C_r = \frac{n!}{r!(n-k)!} = \frac{10!}{(10-4)!} = 210$$

6.2) Contain at most four 1’s

- a) Find bit strings that has a maximum of four 1s, those include bit strings that have: zero 1s, one 1, two 1s, three 1s, or four 1s and add up to the total amount.

$$\binom{10}{0} + \binom{10}{1} + \dots + \binom{10}{4} = 386$$

- b) Number of bit strings of size 10 with at most four 1’s is 386.

6.3) Contain at least four 1’s

- c) Add up the total amount of bit strings that has at least four 1 bits, those include bit strings that has: four 1 bits, five 1 bits, six 1 bits, and so on.

$$\binom{10}{4} + \binom{10}{5} + \dots + \binom{10}{10} = 848$$

- d) Final answer is 848 bit strings have at least four 1 bits.

6.4) Contain an equal amount of 0 and 1.

- e) If we have the same number of zeros and ones, this means that there will be both fives 1’s and 0’s. Thus, we can use the “Combination Formula” to choose the amount of bit strings with five 1’s, which, we will get 252 bit strings.

$${}_n C_r = \frac{n!}{r!(n-r)!} = \frac{10!}{5!(10-5)!} = 252$$

7) Eight people are to be seated in a row.

7.1) How many arrangements are possible if there are no restrictions.

a) There are 8 seats available, thus, there are 8! arrangements.

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40,320$$

7.2) How many arrangements are possible if 2 specific people MUST sit next to each other.

b) Group 2 people up, in that group, there are 2! possible arrangements. We now have 7! ways to arrange the whole group (2-person group + 6 of other people)

c) Using the multiplication principle, multiply the two numbers.

$$2! \times 7! = 10,080$$

7.3) How many arrangements are possible if two specific people must not sit next to each other?

d) Simple subtract the number of total arrangements with the number of arrangements if two people must sit next to each other.

$$40,320 - 10,080 = 30,240 \text{ ways}$$

8) A group of 10 people needs to form a committee.

8.1 How many ways can you select a 5-person committee?

a) Selecting 5 people from a group of 10 people to form a committee can be used with the "Combinations Formula"

$${}^n C_r = \frac{n!}{r!(n-r)!} = \frac{10!}{5!(10-5)!} = 252$$

8.2 How many ways can you select the committee if one specific person must always be included?

b) If one person is always included, this means that the amount of possible choices decreases to 9 – we can still use the "Combinations Formula".

$${}^n C_r = \frac{n!}{r!(n-r)!} = \frac{9!}{5!(9-5)!} = 126$$

8.3 What if two specific people refuse to serve together?

c) Calculate the total number of ways to choose 5 people out of 10 people which is 252.

d) Calculate the number of combinations where two specific volunteers are together by grouping up 2 specific people which leaves us with 9 units (2-person group and the rest of the 8 people)

$$\frac{8!}{4!(8-4)!} = 70$$

e) Subtract the restricted combinations (d) from the total (c)

$$\binom{10}{5} - \binom{8}{4} = 182$$

f) Final answer is 182.

9) We want to form 3 groups from 9 people. If we allow the number of members in each group to be between 2 to 5, how many ways can we form the groups.

a) There are 3 possible group combinations, they are:

- 1) 3, 3, 3
- 2) 2, 3, 4
- 3) 2, 2, 5

b) Calculate the number of ways for each combination.

- 1) 3, 3, 3

$$\frac{\binom{9}{3} \times \binom{6}{3} \times \binom{3}{3}}{3!} = \frac{84 \times 20 \times 1}{3!} = \frac{84 \times 20}{6} = 280$$

- 2) 2, 3, 4

$$\frac{\binom{9}{2} \times \binom{7}{3} \times \binom{4}{4}}{3!} = \frac{36 \times 35 \times 1}{3!} = \frac{36 \times 35}{6} = 210$$

- 3) 2, 2, 5

$$\frac{\binom{9}{2} \times \binom{7}{2} \times \binom{5}{5}}{3!} = \frac{36 \times 21 \times 1}{3!} = \frac{36 \times 21}{6} = 126$$

c) Add up all the total number of possibilities for each combinations.

$$280 + 210 + 126 = 616$$

10) We want to choose 3 fruits out of 6 fruits which are oranges, apples, mangoes, melons, kiwis, or strawberries. However, if we want to get apples, we also have to get strawberries. How many options can we do?

Group up apples and strawberries as one group, leaving with 5 units ([Apples & Strawberries] + 4 fruits). Thus, we can use the multiplication principle to calculate the answer.

$$2! \times 4! = 48 \text{ different ways}$$