

Let $a = 8$, $b = 11$, $c = 3$

$$\begin{aligned} 3x_1 - 4x_2 + 8x_3 &= -1 \\ -3x_1 + 11x_2 + 3x_3 &= 1 \\ 8x_1 + 8x_2 - 11x_3 &= 32 \end{aligned}$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 3 & -4 & 8 & -1 \\ -3 & 11 & 3 & 1 \\ 8 & 8 & -11 & 32 \end{array} \right] \xrightarrow{\substack{R_2 + R_1 \rightarrow R_2 \\ R_3 - (\frac{8}{3})R_1 \rightarrow R_3}} \left[\begin{array}{ccc|c} 3 & -4 & 8 & -1 \\ 0 & 7 & 11 & 0 \\ 0 & \frac{56}{3} & -\frac{97}{3} & \frac{104}{3} \end{array} \right] \xrightarrow{\substack{R_3 - \frac{8}{3}R_2 \rightarrow R_3 \\ 3R_3}} \left[\begin{array}{ccc|c} 3 & -4 & 8 & -1 \\ 0 & 7 & 11 & 0 \\ 0 & 0 & -185 & 104 \end{array} \right]$$

Back substitution.

$$\begin{aligned} -183x_3 &= \frac{104}{-183} \\ x_3 &= -\frac{104}{183} \end{aligned}$$

$$\begin{aligned} 7x_2 + 11x_3 &= 0 \\ 7x_2 + 11x_3 &= -\frac{11}{7}x_3 \\ x_2 &= -\frac{11}{7} \cdot \left(-\frac{104}{183} \right) \\ &= \frac{11 \cdot 104}{7 \cdot 183} \\ &= \frac{1144}{1295} \end{aligned}$$

$$\begin{aligned} 3x_1 - 4x_2 + 8x_3 &= -1 \\ 3x_1 &= -1 + 4x_2 - 8x_3 \\ &= -1 + 4\left(\frac{1144}{1295}\right) - 8\left(-\frac{104}{183}\right) \\ &= \frac{607}{259} \end{aligned}$$

Therefore,

$$\boxed{x_1 = \frac{607}{259}, \quad x_2 = \frac{1144}{1295}, \quad x_3 = -\frac{104}{183}}$$

Geometric interpretation.

The three equations represent three different planes, in which, they are not parallel & not concurrent along a line, i.e. they intersect at a point.

And the point is $\left(\frac{607}{259}, \frac{1144}{1295}, -\frac{104}{183} \right)$

Given system:

$$\begin{cases} 3x_1 - 4x_2 + ax_3 = -1, \\ -3x_1 + bx_2 + cx_3 = 1, \\ ax_1 + 8x_2 - bx_3 = 32, \end{cases} \quad \text{where } a = 8, b = 11, c = 3.$$

$$\Rightarrow \begin{cases} 3x_1 - 4x_2 + 8x_3 = -1, \\ -3x_1 + 11x_2 + 3x_3 = 1, \\ 8x_1 + 8x_2 - 11x_3 = 32. \end{cases}$$

$$\Rightarrow \begin{cases} 3x_1 - 4x_2 + 8x_3 = -1 \\ -3x_1 + 11x_2 + 3x_3 = 1 \\ 8x_1 + 8x_2 - 11x_3 = 32 \end{cases}$$

Turn into augmented matrix:

$$\text{Augmented matrix: } \left[\begin{array}{ccc|c} 3 & -4 & 8 & -1 \\ -3 & 11 & 3 & 1 \\ 8 & 8 & -11 & 32 \end{array} \right]$$

Gauss elimination (by row operations)

Start with the augmented matrix and swap $R_1 \leftrightarrow R_3$ $R_1 \leftrightarrow R_3$ to use an 8-pivot:

$$R_1 \leftrightarrow R_3 \Rightarrow \left[\begin{array}{ccc|c} 8 & 8 & -11 & 32 \\ -3 & 11 & 3 & 1 \\ 3 & -4 & 8 & -1 \end{array} \right]$$

Eliminate x_1 from R_2, R_3

$$R_2 \leftarrow 8R_2 + 3R_1, \quad R_3 \leftarrow 8R_3 - 3R_1$$
$$\Rightarrow \left[\begin{array}{ccc|c} 8 & 8 & -11 & 32 \\ 0 & 112 & -9 & 104 \\ 0 & -56 & 97 & -104 \end{array} \right]$$

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Eliminate x_3 from R_3 :

$$R_3 \leftarrow 2R_3 + R_2 \Rightarrow \left[\begin{array}{ccc|c} 8 & 8 & -11 & 32 \\ 0 & 112 & -9 & 104 \\ 0 & 0 & 185 & -104 \end{array} \right]$$

Back Substitution

$$\text{Back substitution: } \begin{cases} x_3 = \frac{-104}{185} \\ 112x_2 - 9x_3 = 104 \Rightarrow x_2 = \frac{1144}{1295} \\ 8x_1 + 8x_2 - 11x_3 = 32 \Rightarrow x_1 = \frac{607}{259} \end{cases}$$

So

$$\boxed{x_1 = \frac{607}{259}, \quad x_2 = \frac{1144}{1295}, \quad x_3 = -\frac{104}{185}}$$